OPEN EUROPEAN JOURNAL ON VARIABLE STARS
https://oejv.physics.muni.cz
January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964

ESTIMATING ORBITAL PARAMETERS FOR VISUAL DOUBLE STARS

Vanukuri, A.[1]; Banks, T.S.[2,3]; Budding, E.[4]; Rhodes, M.D.[5]; and M. Ersteniuk[2]

1) William Fremd High School, 1000 S Quentin Rd., Palatine, IL 60067, USA, anvith.vanukuri@gmail.com

2) William Rainey Harper College, 1200 W Algonquin Rd, Palatine, IL 60067, USA

3) Nielsen, 675 6th Avenue, New York, NY 10010, USA

4) Carter Observatory, P.O. Box 2909, Wellington, New Zealand

5) Brigham Young University, Provo, UT 84602, USA

Abstract: A Bayesian optimisation technique was applied to estimate orbital parameters for the visual double stars Sirius, α Cen, AGC11AB, BU151AB, BU513AB, BU648, BU883AB, STT38BC, and STF1196AB. These estimates were compared with those adopted by the Washington Double Star catalog, showing good agreement. This indicates the reliability of the method, ahead of its planned use for systems with no prior published orbital parameter estimates.

1 Introduction

Visual double stars make interesting subjects for introductory student research projects in astronomy. The orbits of such stars typically takes decades or longer to complete. Students therefore could make observations of the relative positions of the pair of stars in such doubles, adding to similar observations made by previous researchers perhaps across the generations. An alternative, as in the case of this paper, is to build off the efforts of such observers and try to make model orbital fits to the data they collected. A computational project like this allows the student to build up programming and statistical skills which will bode them well regardless of whether they continue on in astronomical research or enter industry.

The current paper describes work by a high school student implementing program code applying optimisation techniques to historical data for 9 systems, building off the work by Erstenuik et al. (2023) in which more details can be found. A Hamiltonian Markov Chain (HMC) Monte Carlo optimisation technique was applied. An advantage of using a Bayesian technique, such as Markov Chain Monte Carlo (MCMC) method like this, compared to point optimisation techniques is that MCMC explores and then characterizes a distribution by randomly sampling it without requiring knowledge of the distribution's mathematical properties. This allows estimation of how well parameter estimates are defined. Many commonly applied techniques, such as Thieles-Innes (see Alzner 2004), do not provide such 'uncertainties'. MCMC optimisation is not as computationally efficient in reaching point estimates for parameters as some other popular optimisation methods (such as Levenberg-Marquardt and similar) but instead has the advantage of giving insight into how confident those estimates actually are. Yamada et al. (2022) compared basic MCMC techniques (such as "random walk Metropolis-Hastings") with Hamiltonian

Table 1: Background information on the modeled pairs, taken from the Washington Double Star (WDS) catalog. Right Ascension (α) is in hours, minutes and seconds, while declination (δ) in degrees, minutes, and seconds. 'Obs. Years' gives the range of the modeled observations in years. 'Alternative IDs' give alternative names as a convenience for later researchers, 'WDS ID' is the WDS catalog identification, and 'constellation' is self-explanatory.

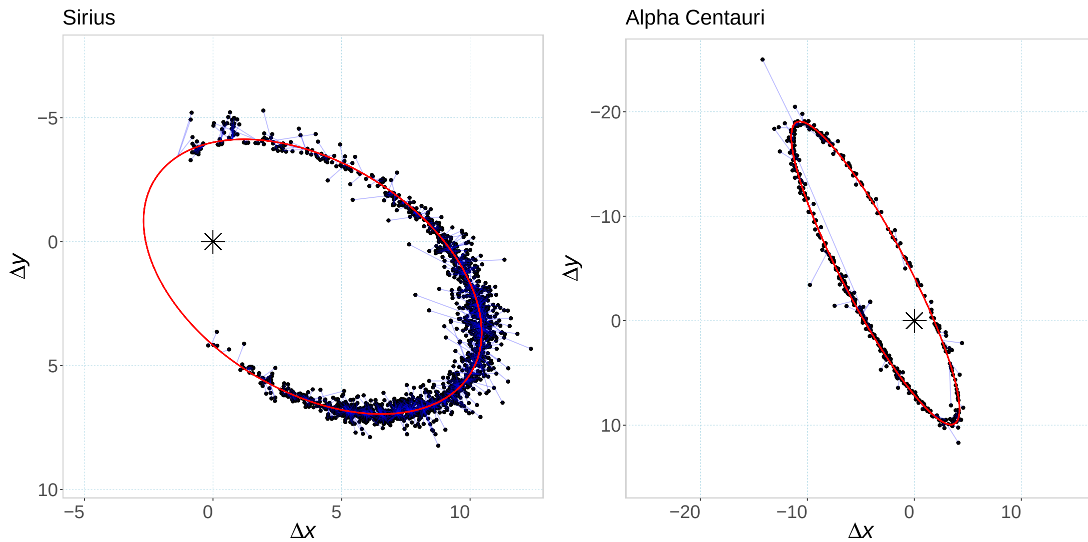
Star	Alternative IDs	WDS ID	Obs. Years	α (J2000))	δ (J2000)	Constellation
AGC1AB	Sirius, HD48915, HIP32349	06451-1643	1862 - 2021	06 50 08.92	−16 42 58.0	Canis Major
RHD1AB	α Cen, HD128620, HIP71683	14396-6050	1752 - 2019	14 39 36.50	−60 50 02.3	Centaurus
AGC11AB	ζ Sge, HD187362, HIP97496	19490+1909	1875 - 2010	19 48 58.65	+19 08 31.1	Sagitta
BU151AB	β Del, HD196524, HIP101769	20375+1436	1873 - 2018	20 37 32.87	+14 35 42.7	Delphinus
BU513AB	48 Cas, HD12111, HIP9480	02020+7054	1878 - 2013	02 01 57.55	+70 54 25.4	Cassiopeia
BU648AB	HD176051, HIP93017	18570+3254	1878 - 2020	18 57 01.61	+32 54 04.6	Lyra
BU883AB	HD30810, HIP22550	04512+1104	1879 - 2021	04 51 12.48	+11 04 05.0	Orion
STT38BC	γ 02 And, HD12534, HIP9640	02039+4220	1843 - 2021	02 03 53.92	+42 19 47.5	Andromeda
STF1196AB	ζ^1 Cnc, HD68257, HIP40167	08122+1739	1825 - 2021	08 12 12.79	+17 38 51.2	Cancer

methods, explaining that HMCs generally require shorter Markov chains to reach convergence compared to basic MCMC methods. This is because HMCs leverage gradient information from the posterior probability distribution function and so are able to transition long distances in the parameter space while still maintaining a higher acceptance ratio. They are therefore more efficient. The acceptance ratio is the probability that a proposed new state in the Markov chain will be accepted as the next state in that chain. In other words, it determines the probability that the chain adopts a new set of parameter estimates based on the comparison between the current state and the proposed state's likelihood according to the target distribution. Higher acceptance ratios mean that the chain is more likely to explore wider ranges in the parameter distributions, which can be important in 'escaping' local minima in the optimisation. See Robert & Casella (2009); Brooks et al. (2011); Gelman et al. (2013) for further information.

The goal of the project was to develop and test using HMC to derive estimates for orbital parameters, using systems with known solutions. Comparison is made with the adopted orbital parameter estimates in the Washington Double Star (WDS) catalog (Mason et al., 2001), showing good agreement between the estimates of this paper and WDS. This lends confidence for future use of the optimization code on data sets with no known solutions.

Double star measurements are not typically made using Cartesian coordinates and are reported, such as in the WDS, using the measures of separation and position angle. The former is the apparent distance between the two stars, while the latter is the angle between north and an imaginary line from brighter star of the pair to the fainter star in a counter-clockwise direction (north to east). See Chapter 14 of Smart (1977) for additional background. In additional to converting the coordinate system, account must be made for the precession of the Earth. For the current analysis, position angles were precessed to the year 2000. The formula for the adjustment in the position angle (in degrees) due to precession is $\Delta\theta_p = -0.0056 \sin\alpha \sec\delta(t-t_0)$, where α is right ascension, δ is declination, t the observation epoch, and t_0 the desired epoch (formula 6.19 of Cocteau, 1981). We made

January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964



(a) Sirius (b) Alpha Centauri

Figure 1: HMC model orbits for Sirius and Alpha Centauri. The model orbits are the red lines, the primary star is indicated with an asterisk symbol (at the origin), the data points with dots, and the blue lines connect the observations with their expected position on the model orbit. North is upwards and east increases to the right, as is the convention in many visual binary papers. The scale of the axes is identical inside a subfigure. See Table 2 (on page 4) for the values of the parameter estimates and their units.

use of the STAN[1] and R[2] programming languages to implement the MCMC optimisation code.

2 Methodology

The orbit of a (visual) binary star system can be described on the xy plane as (Ribas et al., 2002):

$$\Delta x = \frac{a(1-e^2)}{1+e\,cos\nu}[\cos(\nu+\omega)\sin\Omega + sin(\nu+\omega)\cos\Omega\cos i]$$

$$\Delta y = \frac{a(1-e^2)}{1+e\,cos\nu}[\cos(\nu+\omega)\cos\Omega - sin(\nu+\omega)\sin\Omega\cos i]$$

where a is the orbital semi-major axis, e eccentricity, ν true anomaly, i inclination, Ω longitude of the ascending note, and ω the argument of periapsis. Definition of these parameters may be found in Chapter 5 of Smart (1977). These equations were used as the fitting functions for our optimisation code, while least squares (i.e., minimizing the sum of the squares of the residuals) was used as our measure of goodness of fit (for the orbit predictions compared to the actual observations). The STAN/R code then iterated

[1]https://mc-stan.org/
[2]https://cran.r-project.org/

Table 2: Estimates for the orbital parameters and associated uncertainties for systems for Sirius (AGC1AB) and α Centauri (RHD1AB). The row immediately below the star names indicates the source of the estimates: HMC are Hamiltonian Monte Carlo estimates from the current paper, while WDS are from the Washington Double Star catalog. Angles such as ω, Ω, and i are in degrees. P (orbital period) and T (epoch) are in years. a is in arcseconds. Errors are single standard deviations.

Parameter	Sirius HMC	Sirius WDS	α Cen HMC	α Cen WDS
P	50.0200 ± 0.0243	50.128 ± 0.004	79.91 ± 0.01	79.9835 ± 0.0079
a	7.6113 ± 0.0229	7.4957 ± 0.0025	17.66 ± 0.03	17.7695 ± 0.1091
e	0.5894 ± 0.0018	0.5914 ± 0.0037	0.524 ± 0.001	0.5025 ± 0.0057
ω	145.907 ± 0.615	149.161 ± 0.075	232.3 ± 0.1	217.899 ± 0.685
i	134.675 ± 0.367	136.34 ± 0.04	79.32 ± 0.04	79.316 ± 0.211
Ω	44.273 ± 0.402	45.400 ± 0.007	204.75 ± 0.09	205.301 ± 0.3296
T	1994.052 ± 0.071	1994.5715 ± 0.0058	1955.66 ± 0.01	1955.730 ± 0.122

Table 3: Estimates for the orbital parameters and associated uncertainties for the selected systems from Miller & Pitman (1922), using this paper's HMC methodology. The column 'Par' is short for 'parameter'. Units are the same as in Table 2.

Par	AGC11AB	BU151AB	BU513AB	BU648AB	BU883AB	STT38BC	STF1196AB
P	23.16 ± 0.06	26.62 ± 0.03	60.3 ± 0.3	61.1 ± 0.2	16.33 ± 0.09	60.7 ± 0.3	59.66 ± 0.04
a	0.133 ± 0.007	0.447 ± 0.006	0.62 ± 0.02	1.28 ± 0.01	0.184 ± 0.006	0.26 ± 0.01	0.884 ± 0.005
e	0.87 ± 0.05	0.35 ± 0.02	0.31 ± 0.03	0.26 ± 0.01	0.46 ± 0.04	0.89 ± 0.03	0.337 ± 0.008
ω	357 ± 94	195 ± 4	17 ± 11	256 ± 3	263 ± 93	194 ± 9	280 ± 43
i	157 ± 12	61.5 ± 0.9	21 ± 6	114.6 ± 0.6	0 ± 15	118 ± 5	180 ± 6
Ω	343 ± 95	359 ± 1	78 ± 11	48.0 ± 0.6	86 ± 93	103 ± 5	86 ± 43
T	1980.2 ± 0.2	1962.7 ± 0.2	1963.7 ± 0.9	1972.6 ± 0.6	1988.2 ± 0.2	1951.4 ± 0.5	1929.6 ± 0.2

attempting to essentially improve (minimise) the fitting function to reach best fit parameter estimates. A series of MCMC steps is called a chain. Chain lengths were typically several tens of thousands of steps for the modeled systems, allowing good exploration of the parameter space. The data for the modeled systems was voluminous and covered often more than a single orbit. This combination made for well defined orbits. We therefore did not set values for the starting parameters and instead set physical ranges (important for parameters such as angles which 'repeat' after 2π radians) for them instead, allowing the software to select at 'random' the starting estimates. We used uniform priors, indicating a lack of preference (or prior knowledge) for any value inside each range. Future researchers working on more sparse data might choose to use methods, such as the Thieles-Innes, to estimate starting parameters close to a likely global minimum, and perhaps to assume different prior distributions.

3 Results

We started first with Sirius and α Centauri. These are systems with substantial, high quality data sets collected over several orbits. The Sirius data cover from 1862.104 to 2021.142, and the α Cen data from 1752.2 to 2019.6505. We did not make an assessment of the accuracy of the individual measures and attempt to trim to more 'reliable' estimates

OPEN EUROPEAN JOURNAL ON VARIABLE STARS
https://oejv.physics.muni.cz
January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964

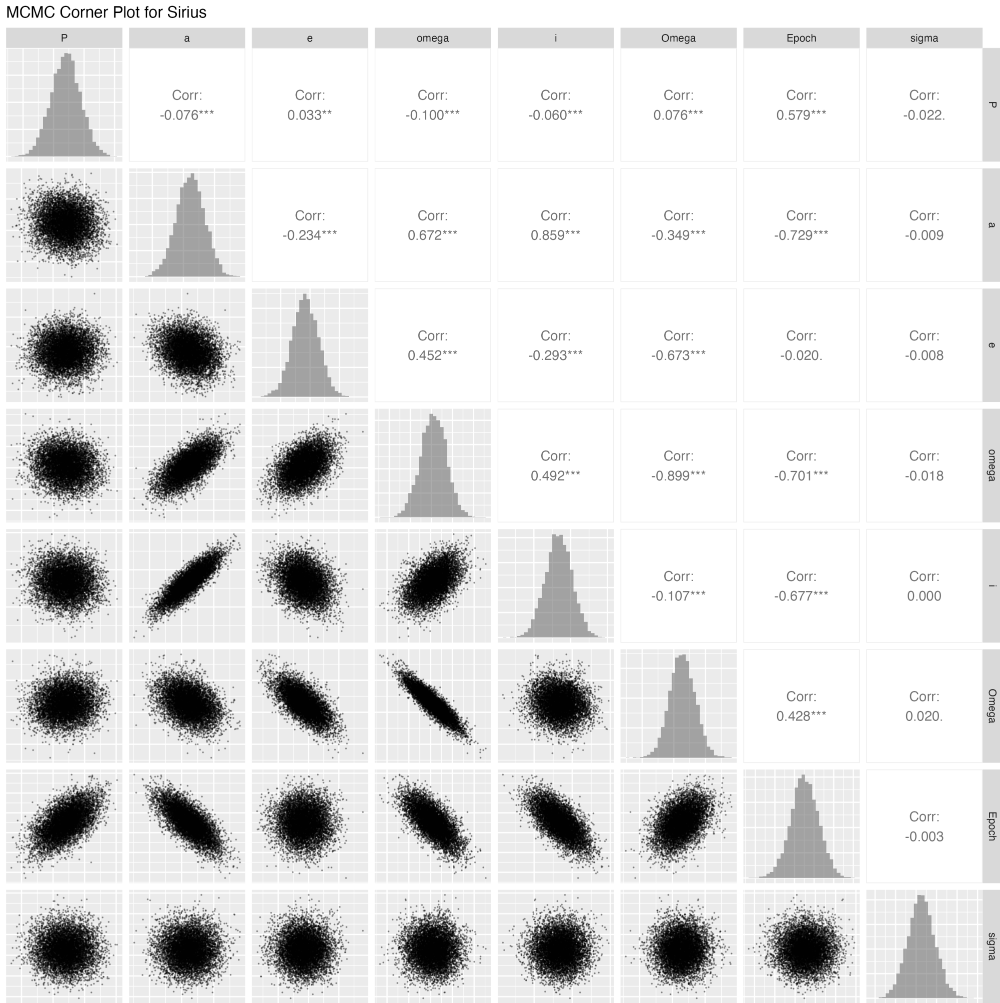


Figure 2: Corner Plot for Sirius. Parameters are as in the main text (bar Ω labeled as 'Omega' and ω as 'omega'). Pearson correlation values ('Corr') are given in the upper right of the figure, bar charts of the chain values for each parameter are shown on the diagonal (from top left to bottom right), and scatter plots showing the chain values for pairs of parameters are shown in the lower left. The figure is based on 8,000 data points from one chain (4 were run), which followed an identical number of prior steps being discarded as 'burn-in'. Sigma is an estimate of the random uncertainty in the observational data.

only. Our goal of the project was to test the program code and optimisation technique, not to necessarily derive the most accurate parameter estimates through judicious consideration and selection of the underlying observations. Such work would make for an interesting follow-up project.

These two systems were modeled ahead of more difficult "second class" systems from Miller & Pitman (1922), which were described as then being "stars whose orbits are less well determined or whose parallaxes have been determined by two or more observers and the results are discordant". We have the advantage of approximately a century's worth of further observations compared to these earlier authors, who unfortunately did not include orbital parameter estimates in their paper. Erstenuik et al. (2023) have already

January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964

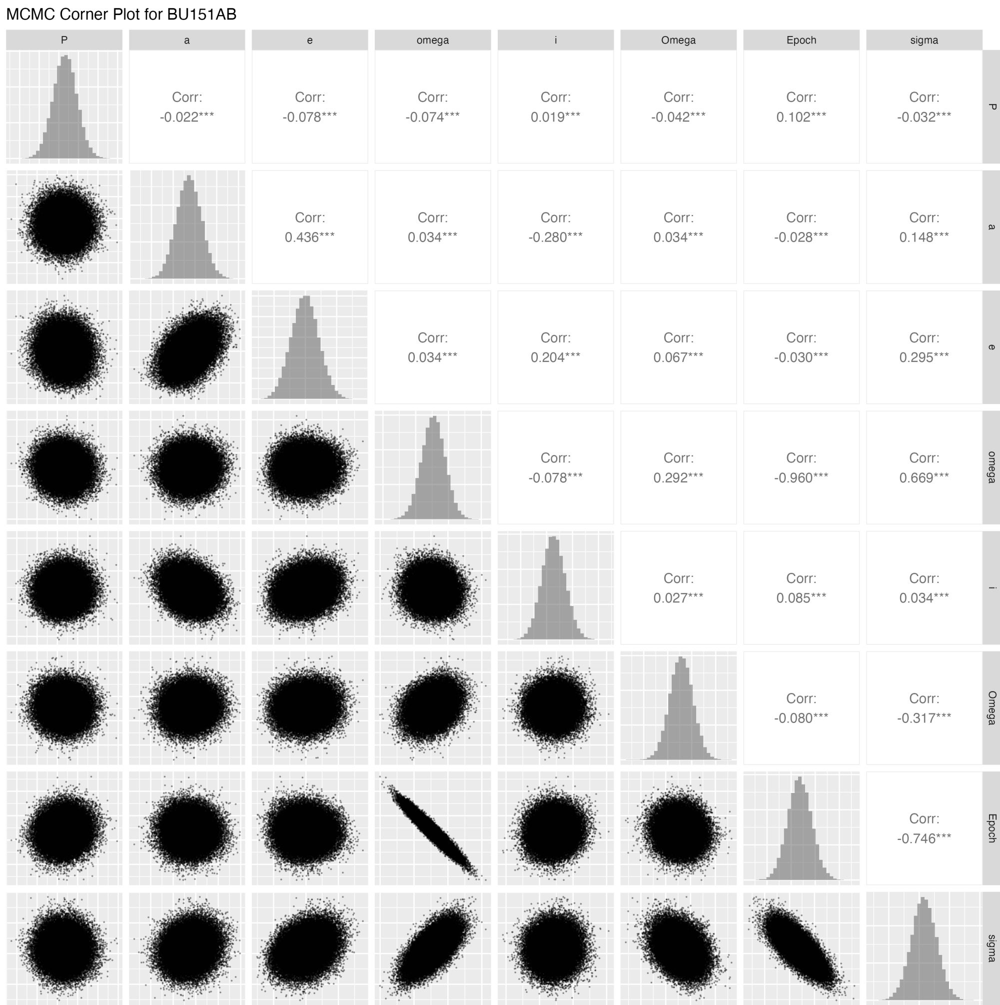


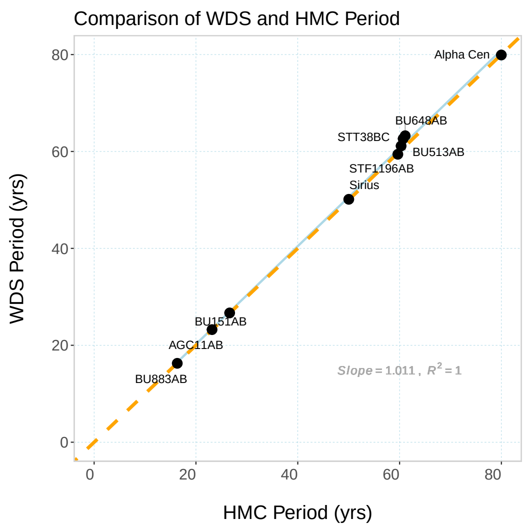
Figure 3: Corner Plot for BU151AB. Parameters and layout are as in Figure 2. The chart is based on 50,000 steps of one chain. Corner plots of the other studied systems are similar and so in the interests of space are not included in this paper, this diagram and Figure 2 being representative.

modeled the "first class" systems from Miller & Pitman (1922), explaining our interest in attempting more difficult systems once we were confident that our modeling technique was reliable.

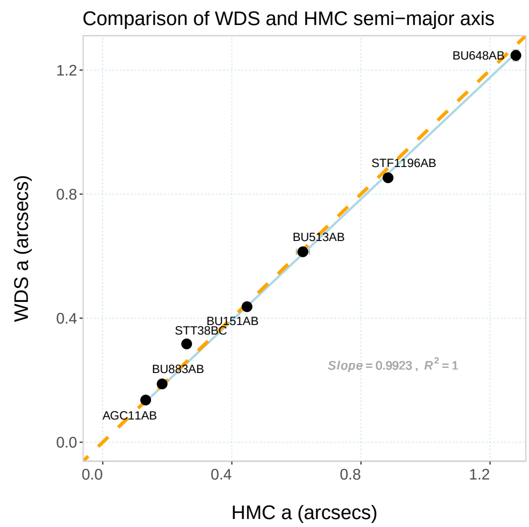
Figure 1 (on page 3) shows the model fits to the WDS data for Sirius and α Cen. Table 2 gives the HMC parameter estimates and uncertainties for the orbital parameters, along with the estimates adopted by the WDS. Overall there is good agreement between the HMC and WDS estimates, although at times outside the formal uncertainties. This gave us confidence that our implementation was operating correctly and we proceeded on to the more difficult systems. Figure 2 (on page 5) is the 'corner plot' of one of the chains used in the optimisation for Sirius, showing distributions of the parameter values explored by the chain.

Table 3 lists the parameter estimates and uncertainties for the selected Miller & Pitman

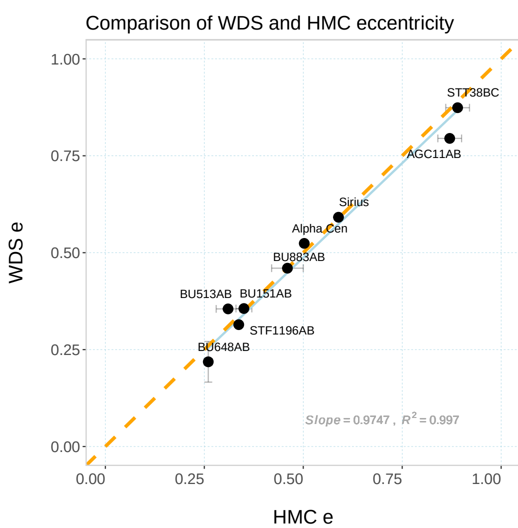
January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964



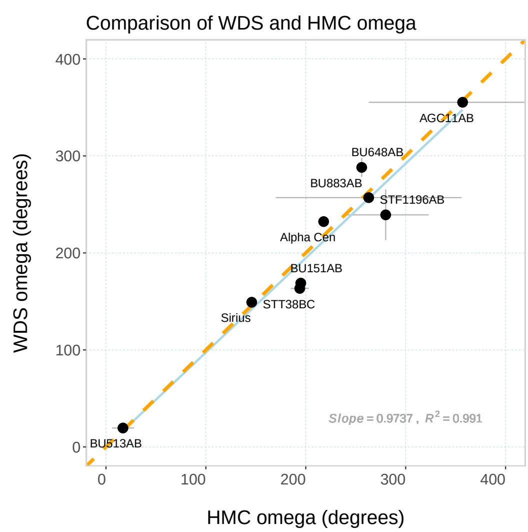
(a) P



(b) a



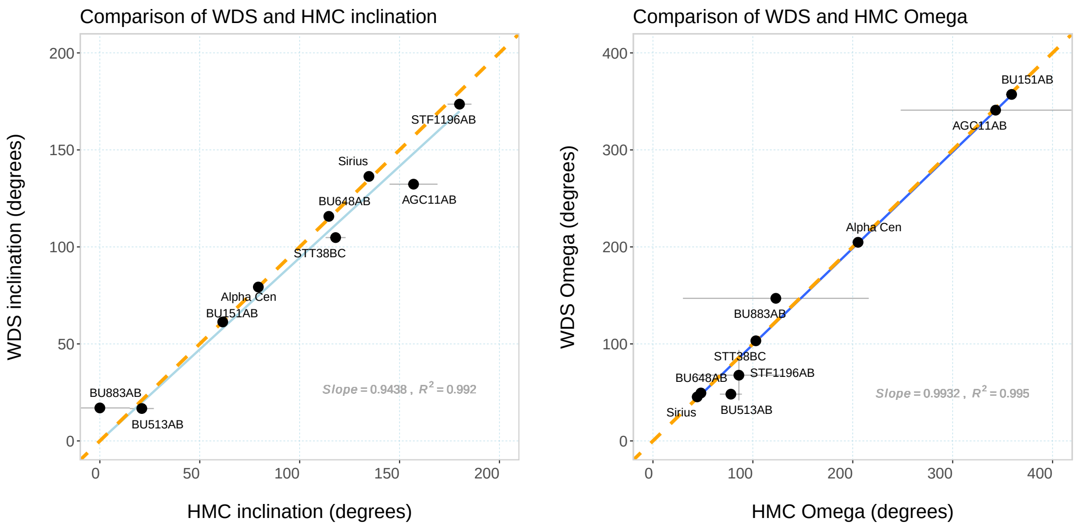
(c) e



(d) ω

Figure 4: Comparison between HMC (this paper) and WDS optimal parameter estimates for P, a, e and ω by system. Linear regressions have been fitted to the data, resulting in best-fit (blue-colored) lines in the charts. The dashed orange lines are those of perfect agreement. The data points fall close to these dashed lines, as do the regression lines, showing that there is good agreement between the two sets of estimates. Due to the larger semi-major axes for α Cen and Sirius, estimates for these two systems were excluded from Subfigure 4b.

OPEN EUROPEAN JOURNAL ON VARIABLE STARS
https://oejv.physics.muni.cz
January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964



(a) i (b) Ω

Figure 5: Comparison between HMC (this paper) and WDS optimal parameter estimates for i and Ω by system. Format is the same as in Figure 4. Again, agreement is good for parameter estimates between this study and other researchers.

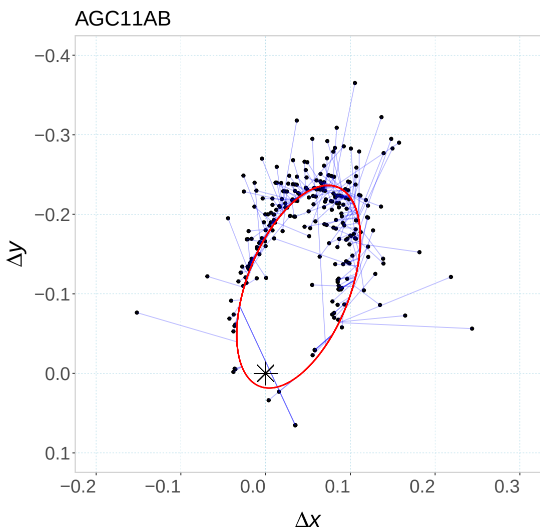
(1922) systems. Figure 3 is a representative corner plot. Figures 4 and 5 plot the HMC estimates versus the WDS values, and uncertainties where available, for all the modeled systems including Sirius and α Cen. The agreement between the HMC results and WDS is good, as indicated by the high values for the coefficients of variation noted on the individual charts. Figures 6 and 7 show the best fit model apparent orbits against the observational data. Despite some systems having widely scattered data, all of the model orbits appear as reasonable fits to the data sets.

4 Conclusions

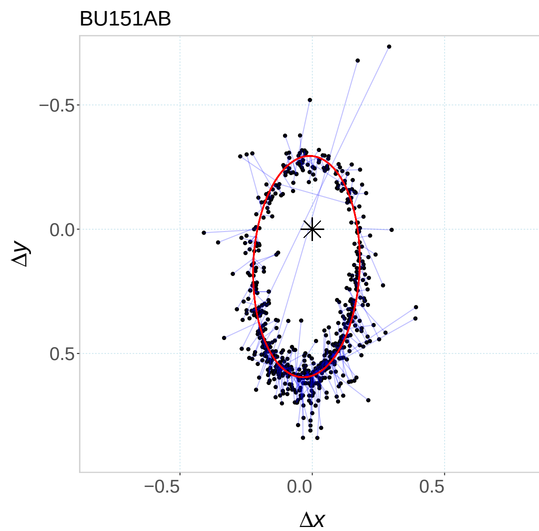
This paper provides updated estimates of the orbital elements and their uncertainties for selected systems taken from the listing of Miller & Pitman (1922), as well as for Sirius and α Cen. It concludes the testing work commenced by Erstenuik et al. (2023) to apply a Bayesian optimisation technique to the problem of estimating orbital parameters of visual double stars. A MCMC-based technique has been shown to produce reliable estimates, with the advantage of also giving insight into the accuracy of those estimates. Our estimates align well with the WDS values, although generally the uncertainties from our HMC-based method are larger (noting that not all the WDS estimates have uncertainties). These comparisons support the reliability of our HMC-based technique, suggesting it can be confidently applied to new systems lacking previous orbital parameter estimates.

The project offered a good REU (Research Experience for Undergraduates) topic, although in this case it was for a high school student. Based on our experience, we recommend the observation and modelling of visual double star data as a good introduction

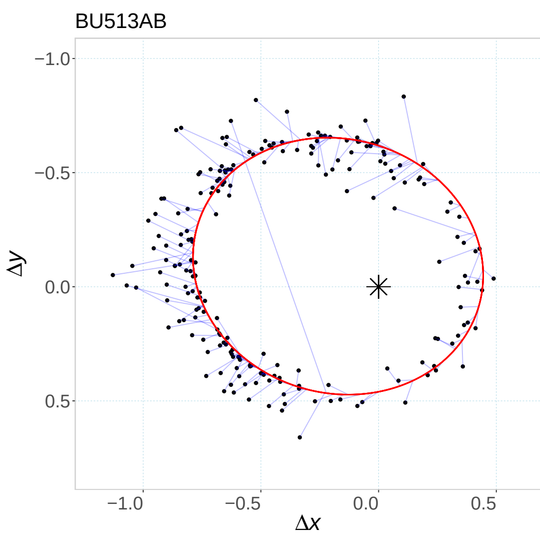
OPEN EUROPEAN JOURNAL ON VARIABLE STARS
https://oejv.physics.muni.cz
January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964



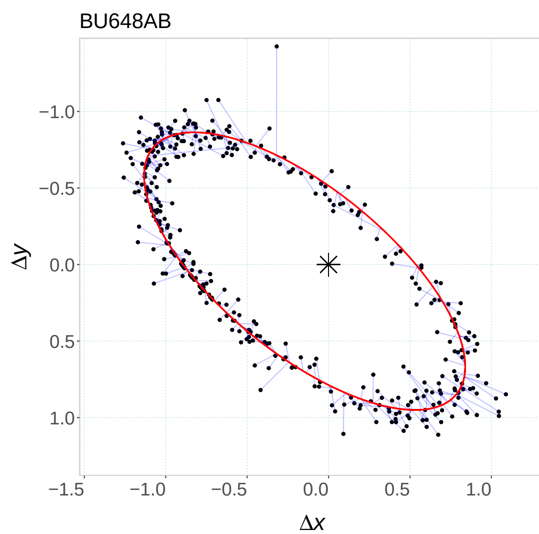
(a) AGC11AB



(b) BU151AB



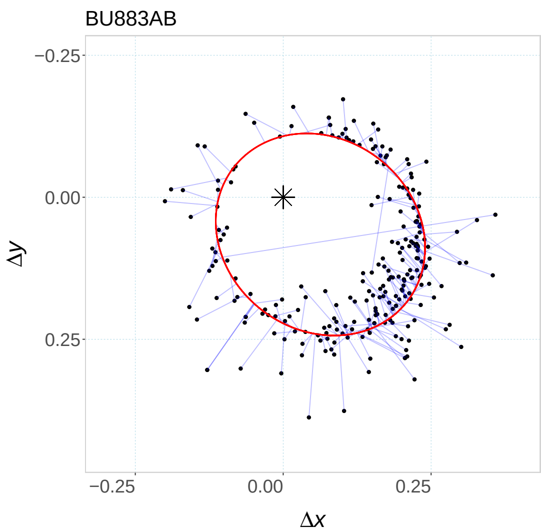
(c) BU513AB



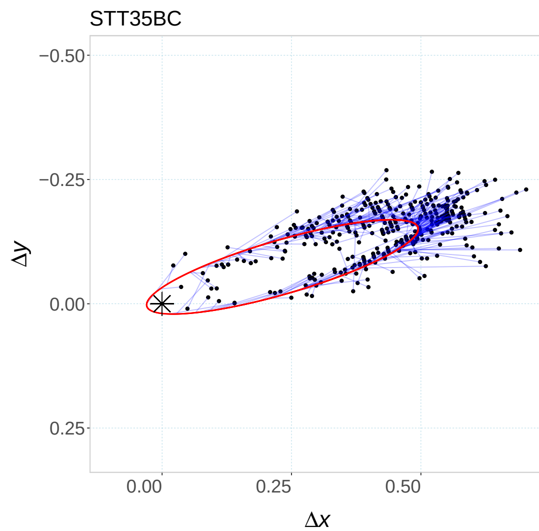
(d) BU648AB

Figure 6: HMC model orbits for AGC11AB, BU151AB, BU513AB, and BU648AB. Layout is the same as in Figure 1. Where the line (from actual to predicted position) passes through the (or close to the) origin this could indicate that the incorrect star was taken as the primary during the observation.

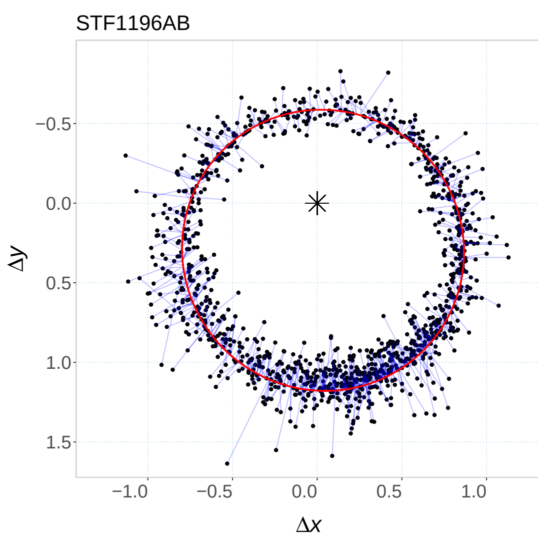
OPEN EUROPEAN JOURNAL ON VARIABLE STARS
https://oejv.physics.muni.cz
January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964



(a) BU883AB



(b) STT38BC



(c) STF1196AB

Figure 7: HMC model orbits for BU883AB, STT38BC, and STF1196AB. Layout is the same as in Figure 6. Axes are on the same scale so as to show the apparent orbits correctly (as they would appear in the sky).

January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964

to astronomical research for motivated high school and undergraduate students. Not only will they learn the scientific method, they develop valuable skills such as programming, presentation, and statistical analysis. Usage of historic data sets can also allow students to feel connected with 'history', the work of earlier generations of astronomers, and if collecting observational data a sense of contributing to future astronomers.

We plan to employ this methodology in our ongoing survey of multiple star systems and recommend it to researchers interested in both estimating orbital parameters and assessing the accuracy of these estimates. Given the increasing number of multiple systems being discovered, we believe that such orbital calculations are a useful addition to the toolkit of variable star astronomer. A recent example would be the group's work about V410 Puppis (Erdem et al., 2022) where the astrometric orbit fit added valuable information to our understanding of this multi-star system. We hope the orbital parameters and uncertainties for the presented test systems will be valuable to double star researchers and serve as a comprehensive record of our methodological validation before applying it to systems with no prior published orbital parameters or dynamical masses.

Acknowledgements: This research has made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory. We are grateful to Dr. Rachel Matson for her assistance extracting the positional data from the WDS catalog for this project. We thank Dr. Štefan Parimucha for his helpful comments and advice as referee, which improved this paper.

References

Alzner, A., 2004, "Orbit Computation", Chapter 8 in Argyle, B., *Observing and Measuring Visual Double Stars*, Springer, DOI: 10.1007/b97664

Brooks, S., Gelman, A., Jones, G.L., & Meng, X-L, 2011, *Handbook of Markov Chain Monte Carlo*, Chapman & Hall/CRC, DOI: 10.1201/b10905

Cocteau, P., 1981, *"Observing Visual Double Stars"*, translated by A. Batten, MIT Press, DOI 10.1604/9780262030779

Erdem, A., Surgit, D., Ozkardes, B., Hardrava, P., Rhodes, M.D., Love, T., Blackford, M.G., Banks, T.S., & Budding, E., 2022, *MNRAS*, **515**, 6151, DOI: 10.1093/mnras/stac2150

Erstenuik, M., Banks, T., Budding, E., & Rhodes, M.D., 2023, *Journal of Astrophysics and Astronomy*, **45**, ID 9, DOI: 10.1007/s12036-024-09997-5

Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A., & Rubin, D.B., 2013, *Bayesian Data Analysis*, 3rd ed., Chapman & Hall/CRC, DOI: 10.1201/b16018

Mason, B.D., Wycoff, G.L., Hartkopf, W.I., Douglass, G.G., & Worley, C.E., 2001, *AJ*, **122**, 3466, DOI: 10.1086/323920

Miller, J.A., & Pitman, J.H., 1922, *AJ*, **34**, 127, DOI: 10.1086/104520

January 2025 DOI: 10.5817/OEJV2025-0259 ISSN 1801–5964

Neal, R.M., 2011, “MCMC Using Hamiltonian Dynamics”, Chapter 5 in Brooks, S., Gelman, A., Jones, G.L., & Meng, X-L, *Handbook of Markov Chain Monte Carlo*, Chapman & Hall/CRC, DOI: 10.1201/b10905

Ribas, I., Arenou, F., & Guinan, E.F., 2002, *AJ*, **123**, 2033, DOI:10.1086/339560

Robert, C.P., & Casella, G, 2010, *Introducing Monte Carlo Methods with R (Use R)*, 1st. ed., Springer-Verlag, Berlin, Heidelberg, DOI: 10.1007/978-1-4419-1576-4

Smart, W.M., 1977, *Text book on Spherical Astronomy*, 7th Edition, Ed: R. Green, Cambridge University Press, DOI: 10.1017/CBO9781139167574

Yamada, T., Ohno, K., & Ohta, Y., 2022, *Earth Planets Space*, **74**, 86, DOI: 10.1186/s40623-022-01645-y