

PERIOD CHANGES OF FOUR CEPHEID VARIABLES

HONTARENKO MYKHAILO¹; MARSAKOVA VLADYSLAVA²; ANDRONOV IVAN³

- 1) Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, Profesora Stanisława Lojasiewicza 11, 30-348, Krakow, Poland, mykhailo.hontarenko@student.uj.edu.pl
- 2) Faculty of Mathematics, Physics and Information Technologies, Mechnikov National University, Dvoriants'ka, 2, 65000 Odessa, Ukraine, v.marsakova@onu.edu.ua
- 3) Dept of Mathematics, Physics and Astronomy, Odessa National Maritime University, Mechnikova St. 34, Odessa, UA-65029, Ukraine, tt.ari@ukr.net

Abstract: We have analysed the AAVSO (2023) photometric (UBVRI) observations of several dozens of Cepheids and W Vir variables. These observations allowed us to find the period changes of four Cepheid variables: ζ Gem, VX Cyg, V Lac, TT Aql. Mean phase light curves were obtained for 3-5 time intervals of observations using trigonometrical polynomial approximations of data in the V-band. VX Cyg shows the period decrease in 400 days interval that is also accompanied by the decrease of asymmetry. It was impossible to find regularities of period changes of other variables within error estimates due to the relatively small amount of observed cycles.

1 Introduction

The term "Classical Cepheid star" is used for pulsating variable stars, which have periods 3-45 days. Cepheids are an important part of the calculation of distances or distance estimation in the Universe, and the study of processes in the pulsating variable stars. Period changes in Cepheids are related to their evolution, mass loss, and inside processes. The light curves of these stars have their special behaviour. They have a fast magnitude increase, and slow decrease, which is related to the thermodynamic process of pulsation. Also, these stars have a period change, which can be related to the process of evolution, irregular luminosity changes or to multi-periodicity, a combination of 2 or more oscillation overlaps Hoffmeister & Richter (1990). A detailed study of the change in periods of 148 galactic Cepheids was made by Csörnyei et al. (2022). They distinguish evolutionary changes in periods (decreasing or increasing), sinusoidal variations associated with the binarity of stars, fluctuations associated with pulsations in overtones, and other internal causes. A detailed study of the change in periods of ζ Gem was also carried out by Engle, Scott (2015) Their studying can provide us with a better understanding of the pulsating process and will help us to find interesting dependencies.

2 Theoretical background

While analyzing a big amount of AAVSO (2023) observations in the UBVRI bands, we have mentioned a period change in four Cepheids, namely (ζ Gem, VX Cyg, V Lac, TT Aql). The periodogram analysis was performed using the software MCV ("Multi-Column Viewer"), the first version of which was introduced by Andronov & Baklanov (2004). Numerous algorithms involved in this program were presented by Andronov (1994); Andronov & Marsakova (2006); Andronov (2020).

We have computed light curves for the observations in different photometric bands, and their characteristics: asymmetry, period, amplitude, initial epoch for the maximum brightness, and characteristics of harmonics. The main form of the trigonometric polynomial fit is

$$m(t) = \sum_{a=1}^m C_a f_a(t), \quad (1)$$

Where we use the basic functions: $f_1(t) = 1$, $f_{2j-1}(t) = \cos(j\omega t)$, $f_{2j}(t) = \sin(j\omega t)$, $\omega = 2\pi/P$, P – is a trial period. The preliminary value of the frequency ω was computed by using the method of periodogram analysis (using the sinusoidal approximation, i.e. the degree of the trigonometrical polynomial $s = 1$), and then the values of ω and C_a were corrected by using the method of differential corrections. The statistically optimal degree of the trigonometric polynomial s using the software FDCN (Andronov 1994). Also, we have constructed $O - C$ diagrams, for which we have taken the moments of maximum brightness from the mean light curves built using the AAVSO (2023) observations in different photometric bands.

3 Data analysis

We have plotted mean light curves in the photometric band V for different time intervals using the observations from the AAVSO (2023). Using the MCV and FDCN, we have determined the period of pulsation, asymmetry of the curve, and degree of the trigonometric polynomial, which are listed in Tables 1-4. We have placed all curves in one figure to visualise the differences between the light curves of different time intervals. The highest curve, number one, has a real (not shifted) magnitude scale. From up to down, the magnitude of the curve can be calculated using the equation: $m = m_{real} + n$, here $n = N - 1$, where N – number of the curve. To create $O-C$ diagram we have used data from: AAVSO (2023); ASAS-SN (2023); BAA (2023); VSOLJ (2023). To maximize the number of points at the $O - C$ diagram for ζ Gem we used the moments of maxima of different authors (Henroteau 1925; Tsarevsky 1967; Erleksova & Irkaev 1982; Turner 1998; Breitfelder, Mérand & Kervella 2016). Furthermore, we used different photometric bands. However, there may be a systematical shift between these moments (maximal luminosity can be achieved in different moments if we observe pulsations at different wavelengths, which correspond to layers with different temperatures). We sometimes calculate the phase difference between phase light curves in different bands (the curves that describe brightness in approximately the same time intervals). In Tables 5-8, moments of maximal

brightness are given. In some cases, we also listed the value of these shifts (jn days) to bring the moments to the moments in the V band.

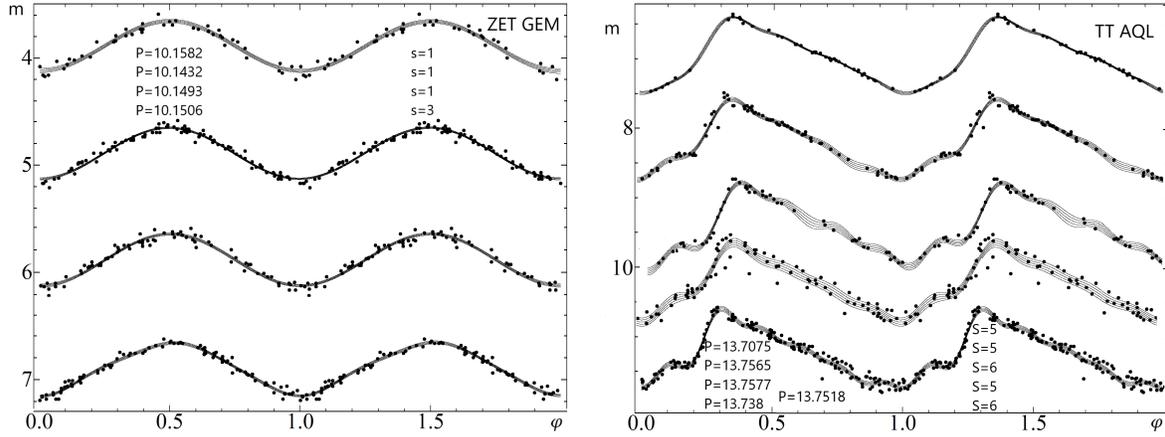


Figure 1: Light phase curves for ζ Gem and TT AQL.

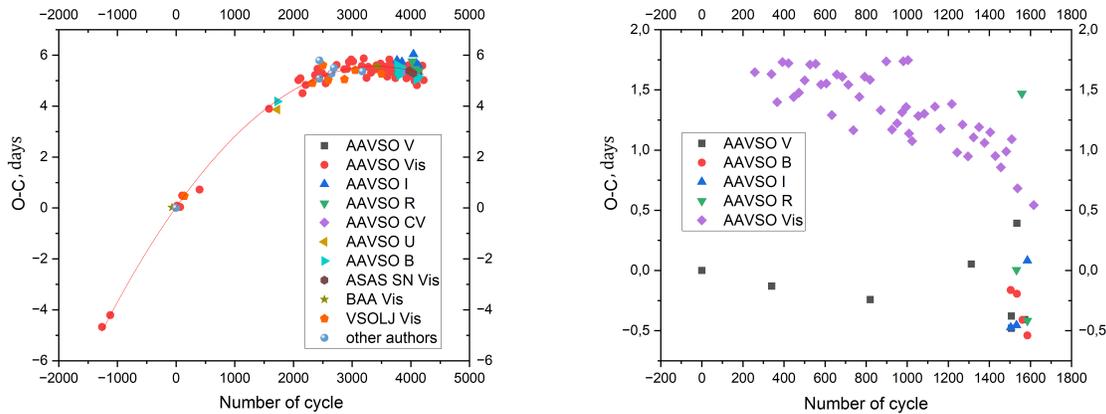


Figure 2: O-C for ζ Gem and TT AQL

ζ Gem showed a strong decrease of the period that is obvious on $O - C$ diagram. The equation of the parabola which describes the character of the period is: $y(x) = B_1x + B_2x^2 + B_3$

Where the most important is B_2 , because $2B_2$ is equal to $\frac{dP}{dE} = \frac{P \cdot dP}{dt}$, We have made the $O - C$ diagram using many sources of data (see description above), and so, according to our observations, the velocity of period change is between $-2.98 \pm 0.07 \frac{sec}{yr}$, which was compared with Engle, Scott (2015) ($3.100 \pm 0.011 \frac{sec}{yr}$), and have shown similar result.

Also, we can see, using the phase curves in V-band, that the decreasing period is followed by the amplitude of pulsation decreasing, so there are maybe changes in the pulsation due to evolutionary changes of the star. For the limited amount of data in the V band, we can suppose changes in asymmetry, with period and amplitude changes.

Time Interval, JD-240000	P , days	$\sigma[P]$, days	A ,m	$\sigma[A]$,m	\mathcal{E}	$\sigma[\mathcal{E}]$	S
57684-57866	10.1582	0.0120	0.463	0.011	0.500	0	1
58035-58238	10.1432	0.0082	0.463	0.011	0.500	0	1
58370-58600	10.1506	0.0039	0.507	0.008	0.509	0.007	3
58731-58966	10.1493	0.0049	0.468	0.005	0.500	0	1

Table 1: ζ Gem’s light curves characteristics. A - amplitude. \mathcal{E} - asymmetry

The star TT Aql showed dependencies between period and amplitude according to the phase light curves. If the star has a period increase/decrease, the same situations can be seen with amplitude. Also, it is interesting that with decreased asymmetry (0.385→0.310), we registered an increase in the degree of the trigonometrical polynomial (5 → 6). Also, interesting behaviour can be seen in the 58215-58433 JD-2400000 cycle of observations. Having an emission spectrum of the star at that moment will help us to understand the star’s evolution better. The $O - C$ diagram has a complicated structure. "Vis" data show an inclined line (the period is constant but slightly smaller than those we took for calculations) Corrected period, according to "Vis" data, is 13.75223(5) days. Nevertheless, O-C points for the V band correspond to a horizontal line or even a parabola. Other filters give us points in close cycles at the end. We can conclude that difference between moments of maxima at different wavelengths changes with time. Thus the shift between radii and temperature curves also can change, causing changes in shape and amplitude (in agreement with noticed ones using phase curve approximations). It would be very interesting to analyse changes with time of radial velocity curves and spectra of this star.

We did not use the shift between moments of maxima in different bands, because it can change between any two wavelengths in this situation.

Time Interval, JD-240000	P , days	$\sigma[P]$, days	A ,m	$\sigma[A]$,m	\mathcal{E}	$\sigma[\mathcal{E}]$	S
41806-42001	13.7505	0.0062	1.072	0.008	0.345	0.007	5
53530-56474	13.7565	0.0006	1.075	0.020	0.363	0.008	5
57876-58047	13.7577	0.0053	1.165	0.024	0.364	0.007	5
58215-58433	13.738	0.0190	0.968	0.028	0.385	0.014	5
58573-59133	13.7518	0.0019	1.092	0.019	0.310	0.006	6

Table 2: Light curve characteristics of TT AQL

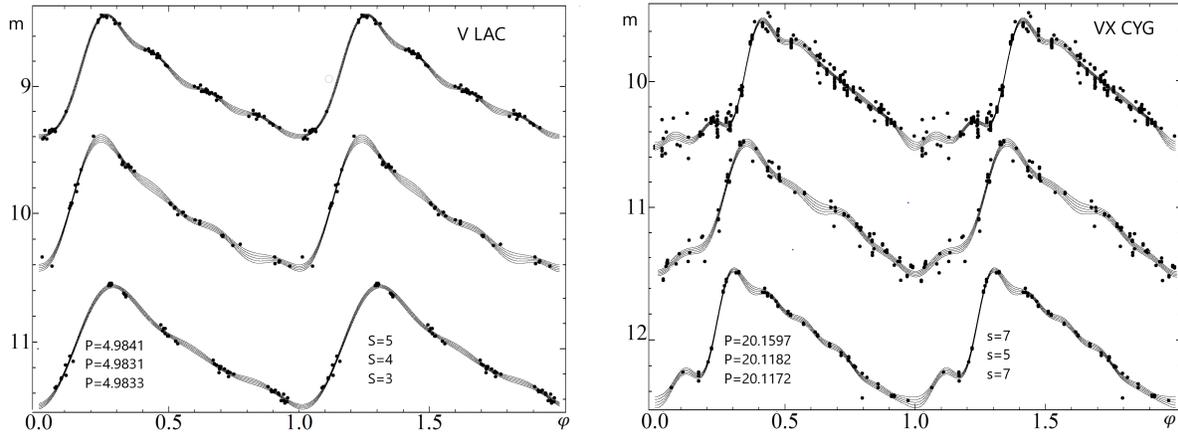


Figure 3: Light phase curves of V LAC and VX CYG.

Time Interval, JD-240000	P , days	$\sigma[P]$, days	A ,m	$\sigma[A]$,m	\mathcal{E}	$\sigma[\mathcal{E}]$	S
36735-37517	4.9841	0.0002	0.970	0.008	0.258	0.012	5
57346-57494	4.9831	0.0003	1.005	0.018	0.239	0.007	4
57534-57702	4,9833	0.0006	0.956	0.009	0.289	0.009	3

Table 3: V LAC light curves characteristics

Time Interval, JD-240000	P , days	$\sigma[P]$, days	A ,m	$\sigma[A]$,m	\mathcal{E}	$\sigma[\mathcal{E}]$	S
57298-57344	20.160	0.028	1.006	0.011	0.413	0.007	7
57346-57494	20.118	0.015	1.048	0.015	0.353	0.007	5
57534-57702	20.117	0.023	1.023	0.010	0.303	0.016	7

Table 4: VX Cyg light curves characteristics

Variable V Lac shows oscillation behaviour on change curves characteristics, but we can see a decrease in polynomial degree. That potential can be used to systematize Cepheid with the same behaviour, and we can suppose that Cepheids have more harmonic signals with their age. Unfortunately, we do not have enough points to make any conclusions with the $O - C$ diagrams, but that points can be used in other work, which will help to receive results. We did not use the shift between moments of maxima in different bands, because all of them were obtained in significantly different cycles.

The light curve of VX Cyg (as well as the light curve of TT Aql) has a bump on the ascending branch. The bumps on Cepheids light curves were noticed for the first time by Hertzsprung (1926) and then analysed by Kukarkin & Parenago (1936). Their position depends on period values: bumps appear on descending branch for nearly six days and move to maximum brightness with an increase in the period and then ascending branch at a period of more than ten days. This phenomenon was called then as Hertzsprung's progression. Bono, Marconi & Stellingwerf (2000) used nonlinear hydrodynamical models for

theoretical investigation on the pulsation behaviour of Bump Cepheids. Their modelling of light and radial velocity curves allows them to obtain a more precise period value in the center of Hertzsprung's progression (where the bump transits the maximum position and goes from the descending to ascending branch of the light curve). Also, they show that the wave, which produces the bump on ascending branch (for periods longer than 10 days), is the same as the maximum for Cepheid with periods less than ten days).

For VX Cyg, we can suppose an increase in the period using the O-C diagram. For three points in the Vis band corresponding to: 57316.86137, 57356.61462, 57578.32918, we have calculated the phase shift between maxima in V and Vis bands because those maxima obtained in the same cycles as V bands ones. Moreover, that shift was added to every Vis maxima point. The average phase shift is 0.1244 with a period of 20.1327 days. Using that points, we have calculated the velocity of period change as $4.7(9) \frac{sec}{yr}$.

According to mean phase lights curves, variable star VX Cyg showed changes in the properties of the bump on an ascending branch (followed by the oscillation of trigonometrical polynomial degree for approximation). Also, we can see that asymmetry decreased with time (Table 4, Fig. 3). All of these features could be connected with the period increasing (thus, the variable could go along Hertzsprung's progression). Nevertheless, a more detailed study of the light curve during the long time interval is necessary for a sure conclusion.

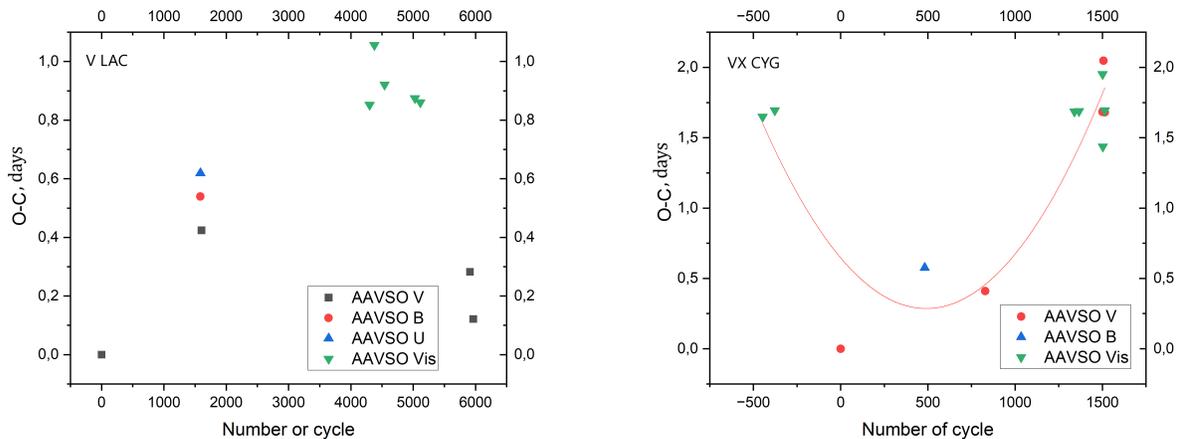


Figure 4: O-C for V LAC and VX CYG

4 Results

According to our calculations, we have received light curves of the stars with their characteristics like period, asymmetry, degree of the trigonometrical polynomial, and conclude that: 1. ζ Gem showed its period decrease -2.98 ± 0.07 s/yr. Also, the star showed a dependence between amplitude and period.

2. TT Aql showed a complex shape of the $O - C$ curve (if we take into account moments of maximal brightness at different wavelengths), more likely due to changes in the shape of the pulsating curve. Analyzing the curve in the V band, we noticed a decrease in the asymmetry value with time ($0.385 \rightarrow 0.310$) which is followed by an increase in the degree of the trigonometrical polynomial fit ($5 \rightarrow 6$).

3. For V Lac, we received some points for the $O - C$ diagram, which can help study that star. Also, we noticed that the star signal became more harmonic for a big period of the star's life from $5 \rightarrow 3$ for 36735-57702 JD.

4. VX Cyg shows the asymmetry decrease in 400 days intervals and possible period increase with the rate of $4.7(9) \frac{sec}{yr}$. The studying of properties of bumps at the ascending branch requires more precise data for analysis

5. TT Aql and VX Cyg show prominent bumps at the ascending branch of the light curves, and their evolution can be related to the period changes detected by using $O - C$ curves.

5 Summary

We analysed the AAVSO (2023) photometric (UBVRI) observations of several dozens of Cepheid and W Vir variables. These observations allowed us to find the period changes in four Cepheid variables: ζ Gem, VX Cyg, V Lac, TT Aql. Mean phase light curves were obtained for 3-5 intervals of observations by using trigonometrical polynomial approximations of data in the V-band (see Figures and Table). We received characteristics of the star's light curves and noticed some dependencies (period-amplitude, asymmetry-period, degree of a polynomial - time of life), which can be found the same in the other stars, which will help us to better systematize Cepheids and understand their nature.

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APPENDIX

Table 5: Moments of maxima for ZET GEM

AAVSO V	AAVSO B	AAVSO I	AAVSO R	AAVSO U	VSOLJ Vis	AAVSO CV
57772.0275	58847.5241	55143.5532	55285.0724	34416.14004	18335.77458	56320.52584
58157.4988	34416.4579	55285.4318	55569.5797	-	40527.23804	55995.75694
58867.9143	55143.3845	55569.6337	55985.6509	-	42314.26939	-
58482.4541	55285.2182	55985.911	57772.2638	-	43237.29719	-
-	55569.2556	57975.5354	58289.5935	-	46048.76818	-
-	55985.6449	58472.5104	58847.7441	-	47906.50726	-
-	57771.8618	58847.9469	-	-	52463.524	-
-	58147.3325	-	-	-	-	-
-	58482.0128	-	-	-	-	-

From ASAS SN Vis: **57264.41219** and **57954.48527**

From BAA Vis: **16203.91496** and **51702.61117**

AAVSO Vis		
4100.8966	49398.95475	54554.38959
5552.75345	49854.91903	54848.51978
17107.29574	50159.84129	54919.93553
17614.71972	50646.79807	55102.66185
17970.40722	51205.17768	55275.00874
20995.22518	51286.3472	55579.68051
32985.0711	51580.84789	55894.16137
38091.44587	51946.14554	56005.82646
38436.60946	52301.53144	56279.94348
38801.40177	52687.02932	56655.50885
39349.80401	53022.0289	56756.63493
40974.13321	53336.72932	57010.63951
41380.35944	53407.64399	57112.38156
41725.15931	53458.08232	57457.39883
42182.00861	53498.7481	57720.98303

Table 6: Moments of maxima for TT AQL

AAVSO V	AAVSO B	AAVSO I	AAVSO R
41912.7663	57964.9515	57923.5914	57923.902
55283.1062	58336.5306	58308.7579	58309.2174
57964.8504	58707.7074	59038.3268	58668.3199
58337.1165	59037.7034	-	59037.8256
58859.0273	58337.1165	-	-
AAVSO Vis			
40486.14667	45573.36072	50318.60827	54334.91358
40786.60715	45944.49129	50648.82445	54692.78094
41581.66357	46274.95331	50731.78498	55063.91067
41900.77202	46646.32722	50923.97873	55435.46231
42285.68804	47045.16809	51048.16273	55806.94075
42643.65886	47388.67119	51116.33549	56164.44691
43042.55418	47787.85329	51322.59741	56549.68486
43386.15489	48145.65783	51735.46271	56893.36885
43743.82841	48503.26968	52134.38536	57264.66874
44129.08029	49218.29211	52849.72078	57622.43677
44486.84908	49590.08999	53220.93065	57993.9327
44858.24955	49974.67512	53977.67808	58378.67505
45215.7141	-	-	59465.203

Table 7: Moments of maxima for V LAC

AAVSO V	AAVSO B	AAVSO U	AAVSO Vis
36889.6162	36800.0364	36805.0988	50329.46489
58620.6178	-	-	50713.36707
58356.6695	-	-	51530.46216
28901.285	-	-	53957.18503
-	-	-	54385.71574

Table 8: Moments of maxima for VX CYG

AAVSO V	AAVSO B	AAVSO Vis
57314.0911	43783.642	36797.7619
57415.1233	-	18096.57532
57575.8141	-	19505.90144
27093.23	-	54074.22325
-	-	54597.78511
-	-	57316.86137
-	-	57356.61462
-	-	57578.32918